ACI STRUCTURAL JOURNAL TECHNICAL PAPER

Title no. 100-S13

N-M-V Interaction Domains for Box and I-Shaped Reinforced Concrete Members

by Antonino Recupero, Antonino D'Aveni, and Aurelio Ghersi

This paper proposes an approximate physical model to evaluate the N-M-V interaction resistance domains for box and I-shaped concrete cross sections. The model subdivides the concrete beam in layers, with nearly constant stress fields, and determines the internal forces satisfying equilibrium of the cross section, applying the static theorem of plasticity (lower bound theorem). The resisting contribution of the web longitudinal reinforcements is also considered by including stress fields of variable inclination on the longitudinal element direction. The proposed model leads to a conservative evaluation of bearing capacity and permits to numerically obtain the N-M-V interaction resistance domains of the indicated cross sections. The obtained N-M-V failure surfaces are in good agreement with the experimental evidences of tests performed by P. Regan and H. Rezai-Jorabi and by J. R. Robinson and J. M. Demorieux.

Keywords: flexural strength; reinforced concrete; shear strength; steel.

INTRODUCTION

Structural reinforced concrete elements are often subjected to both transversal and axial loads that simultaneously generate in the cross section axial force *N*, bending moment *M*, and shear force *V*; their interaction effects provide the beam failure mode.

International building codes of practice do not appropriately consider the *N-M-V* interaction effects and prescribe design formulas for axial force and bending moment separately from those referred to shear strength. The latter are classically obtained by superposing the shear capacity of the truss model to that of the beam without stirrups, ignoring any stirrup influence on concrete failure mechanisms.

In the last 40 years, theoretical and experimental investigations have clarified the most important aspects of shear failure.

Beams without web reinforcement[1,2] may fail without attaining their full bending capacity and in a brittle way because of shear (shear failure). The shear resistance is given by two different mechanisms, the beam action and the arch action, depending on the longitudinal reinforcement ratio $\rho = A_s/A_c$ and the shear span-depth ratio $a/d = M/Vd$.

The presence of web reinforcement does not fundamentally change the mechanism (beam and arch action) but allows an additional shear resistance related to the web reinforcement ratio ($\rho_w = A_w/b_w s$).

Recently, the older models introduced by Ritter and Mörsch, based on the truss analogy (with a 45 degree angle for the diagonal compression strut), have been replaced by a new model[3] that considers stress fields: compressive stresses in concrete, and uniformly distributed tensile stresses, corresponding to the action of the stirrups. In this approach, the angle ϑ of compressive stresses may be different from 45 degrees; as a matter of fact, it varies as shear force increases, after the yielding of web reinforcements. This model has been enhanced[4] to include a further stress field corresponding to longitudinal reinforcements in the web.

In recent years, the contribution of axial force *N* to the *N-M-V* interaction effects has been analyzed, mainly in investigations on the performance of bridge piers located in seismic areas. Design formulations either strictly inspired by experimental evidences or based on analytical empirical and semi-empirical models[5,6] have been proposed on this topic. The comparison of experimental evidences and results of the proposed approaches, however, have shown that no satisfactory model for *N-M-V* interaction has yet been developed. A recent paper[7] proposed a generalization of the stress field approach that also accounted for the effect of axial force. The effectiveness of this proposal was strongly limited by the fact that the compressed chord and the tension stringer were concentrated along lines, as in the classical truss model. A following paper[8] by the same authors modified the previous approach to allow the presence of further stress fields with variable depth, corresponding to the compressed chord and tension stringer. This proposal was limited to rectangular cross sections and did not allow taking into account the presence of web longitudinal reinforcements.

This paper generalizes the variable-depth stress-field model to box and I-shaped cross sections subjected to axial force *N*, bending moment *M*, and shear force *V*. The proposed model also includes a stress field corresponding to longitudinal reinforcements in the web. Based on the models, the authors obtain *N-M-V* interaction domains and explore the favorable and unfavorable effects of the shear *V* and axial force *N* on bending moment *M*. The interaction domains give a convenient graphic design-verification procedure able to take into account the exact beam failure mode in the presence of *N*, *M*, *V* internal actions.

Finally, the comparison between the proposed model and the results of a testing campaign[9] shows that the model provides a good interpretation of the results of the cited tests.

RESEARCH SIGNIFICANCE

The interaction of axial force, bending moment, and shear force has been examined in few papers, which have not proposed, up to now, a model able to compare in a reliable way with the experimental evidence. For this reason, most codes prescribe to separately consider the effect of these internal actions. This paper generalizes the variable-depth stress-field approach, allowing consideration of the effect

ACI Structural Journal, V. 100, No. 1, January-February 2003.
MS No. 02-081 received March 6, 2002, and reviewed under Institute publication policies. Pertinent discussion will be published in the November-December 2003 *ACI Structural Journal* if received by July 1, 2003.

Antonino Recupero *is a research assistant in the Department of Construction and Advanced Technologies at Messina University, Italy. His research interests include shear in reinforced concrete members, stress fields, and interaction problems related to design procedures.*

Antonino D'Aveni *is an assistant professor of structural engineering at the University of Catania. His research interests include numerical problems in structural dynamic.*

Aurelio Ghersi *is a professor of structural engineering and Coordinator of the Structural Engineering Section of the Department of Civil and Environmental Engineering (DICA) at the University of Catania. His research interests include seismic design of reinforced concrete and steel structures and cold-formed metal structures.*

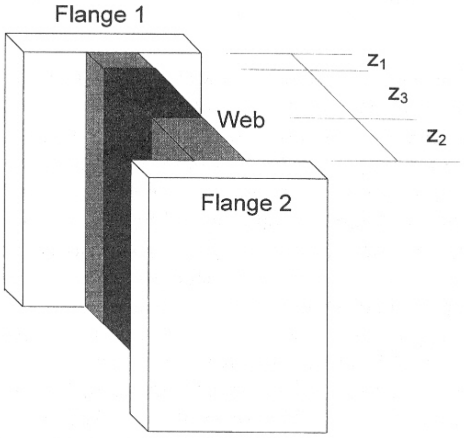

Fig. 1—Layered structural element.

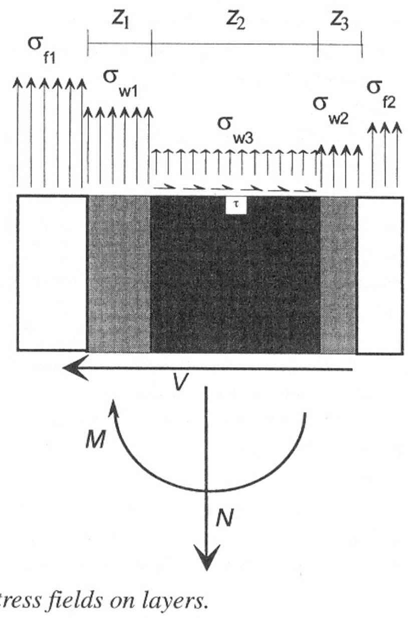

Fig. 2—Stress fields on layers.

of longitudinal and transversal reinforcement in the web and analysis of I-shaped cross sections subjected to *N, M, V* by interaction domains that provide a powerful tool for optimizing the disposition of reinforcements and for maximizing the cross-section strength.

Proposed model

Because of the simultaneous presence of axial force *N*, bending moment *M*, and shear force *V*, the actual stress fields trend in the cross section of a structural element is complex and it is difficult to propose an analytical model that may determine their exact distribution. Nevertheless, it is less difficult to evaluate the ultimate resistance of the structural element when the following assumptions are made:

- The longitudinal and transverse reinforcements are subjected only to axial forces, including forces due to bending moments (dowel action is negligible); their action is expressed by distributed stress fields, provided that they are shortly spaced; and the stress fields are assumed uniform, based on the theory of plasticity of steel structural elements;
- The chord and web concrete are subjected only to compressive stress fields, once again assumed uniform;
- The uniform web stress field has a ϑ degree angle on the longitudinal direction, which may differ from 45 degrees because of the actions transmitted along the shear fractures; there is no reason why this angle should remain constant; and
- The failure mode of the structural element occurs for concrete crushing or for reinforcements yielding or both.

By these assumptions, the analytical model of the structural element is an evolution of the truss model that replaces all the components (compressed chord, tension stringer, strut, and tie) by uniform stress fields.

The general criterion adopted consists of dividing the basic structural element in several layers (Fig. 1) having depth z_i not defined a-priori, subjected to uniform distributions of stresses (normal σ, shear τ, or both), so as to obtain the equilibrium with the internal actions *N, M, V* as a whole (Fig. 2). This approach, typical for the design of in-plane loaded plates, has proven to be equivalent to the stress fields approach.[8] Any number of subdivisions may be assumed, provided that the basic structural element is in equilibrium. In particular, the analyzed box or I-shaped cross section is subdivided in five layers (one for each flange, three for the web); the flanges and the nearby portions of web are subjected only to normal stresses, equivalent to the action of compressed chord and longitudinal reinforcements; and the central web layer is subjected to both shear and normal stress components, equivalent to the stress fields of strut and web transversal and longitudinal reinforcements.

The equilibrium conditions are obtained as demonstrated for a three-layer model,[10] imposing that the response is governed by the weakest stress-field failure (for more detail, refer to Appendix). The design values of CEB-FIP Model Code 1990[11] for steel yielding f_{yd} and for concrete strength f_{cd1} and f_{cd2} are used.

- In the central web layer

$$(-\sigma_{w3}-\rho_{wl}f_{yd})\tan\vartheta \le \tau \le (-\sigma_{w3}+\rho_{wl}f_{yd})\tan\vartheta \qquad (1a)$$

$$\tau \le \rho_{wt}f_{yd}\cot\vartheta \qquad (1b)$$

$$\tau \le f_{cd2}\sin\vartheta\cos\vartheta \qquad (1c)$$

- In the lateral web layers

$$-f_{cd1}-\rho_{wl}f_{yd} \le \sigma_{w1} \le \rho_{wl}f_{yd} \qquad (2a)$$

$$-f_{cd1}-\rho_{wl}f_{yd} \le \sigma_{w2} \le \rho_{wl}f_{yd} \qquad (2b)$$

- In the top and bottom flange layers

$$-f_{cd1}-\rho_{f1}f_{yd} \leq \sigma_{f1} \leq \rho_{f1}f_{yd} \tag{3a}$$

$$-f_{cd1}-\rho_{f2}f_{yd} \leq \sigma_{f2} \leq \rho_{f2}f_{yd} \tag{3b}$$

The stress fields acting at failure on the cross section must be equivalent to the resisting internal actions N_{Rd}, M_{Rd}, and V_{Rd}. The following relations can be written:

$$N_{Rd} = \int_S \sigma dS = \sigma_{f1}\int_{S_{f1}} dS + \sigma_{w1}\int_{S_{w1}} dS + \sigma_{w3}\int_{S_{w3}} dS + \tag{4a}$$

$$\sigma_{w2}\int_{S_{w2}} dS + \sigma_{f2}\int_{S_{f2}} dS$$

$$M_{Rd} = \int_S \sigma y dS = \sigma_{f1}\int_{S_{f1}} ydS + \sigma_{w1}\int_{S_{w1}} ydS + \tag{4b}$$

$$\sigma_{w3}\int_{S_{w3}} ydS + \sigma_{w2}\int_{S_{w2}} ydS + \sigma_{f2}\int_{S_{f2}} ydS$$

$$V_{Rd} = \int_S \tau\, dS = \tau\int_{S_{w3}} dS = \tau b_w z_3 \tag{4c}$$

In Eq. (4a) and (b), the integrals related to the areas S_{f1} and S_{f2} give numerical values because the depth of flanges is fixed. On the contrary, the integrals related to the areas S_{w1}, S_{w2}, and S_{w3} can be expressed in function of the depth of the web layers z_1, z_2, and z_3, which may vary according to the following static and geometrical conditions

$$z_3 \geq z_{3min} \tag{5a}$$

$$z_1 \geq 0 \qquad z_2 \geq 0 \qquad z_1 + z_2 + z_3 = h_w \tag{5b}$$

In particular, Eq. (5a) states that the central web layer depth must be sufficient to bear shear stresses. Its minimum value, which depends on concrete strength f_{cd2} and on transversal web reinforcement mechanical ratio ω_{wt}, is given by[4]

$$z_{3min} = \frac{V_{Sd}}{f_{cd2}b_w\sqrt{\omega_{wt}(1-\omega_{wt})}} \quad \text{if } \omega_{wt} \leq 0.5 \tag{6a}$$

$$z_{3min} = \frac{2V_{Sd}}{f_{cd2}b_w} \quad \text{if } \omega_{wt} > 0.5 \tag{6b}$$

Design of reinforcements and *N-M-V* interaction domains

The conditions described in the previous section, managed according to nonlinear programming procedures, may be used: (a) for evaluating the reinforcements of the cross section, when the geometry and the design values N_{Sd}, M_{Sd}, and V_{Sd} are given; and (b) for obtaining resistance domains of the cross section, when its geometry and reinforcement ratios are defined.

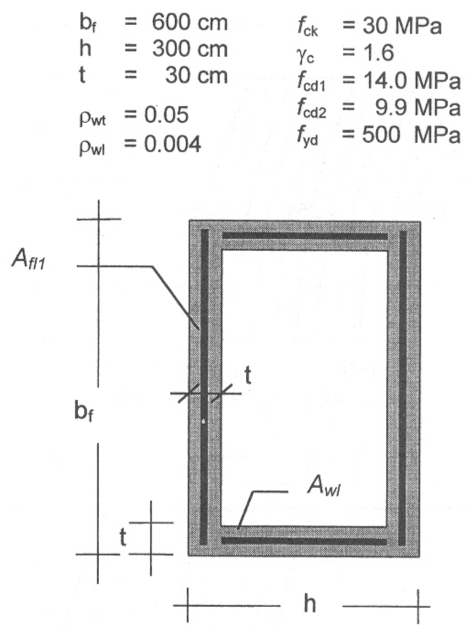

Fig. 3—Cross section of bridge pier.

In case (a), it is possible to proceed according to the following steps:

1. Define values of transversal and longitudinal reinforcement of the web;
2. Evaluate the minimum web depth z_{3min} by Eq. (6); if $z_{3min} > h_w$, it is not possible to proceed because the whole web is not able to withstand shear stresses, it is necessary to start again with a larger value of ω_{wt} (or, if it is already higher than 0.5, no solution is possible with this cross section);
3. Assign trial values of z_1, z_2, and z_3, respecting conditions (5);
4. Evaluate the shear stress τ by Eq. (4c);
5. Assign the angle ϑ in the range defined by Eq. (1b) and (1c);
6. Assign trial values of σ_{w3}, σ_{w1}, and σ_{w2} in the ranges defined by Eq. (1a), (2a), and (2b), respectively;
7. Evaluate σ_{f1} and σ_{f2} by Eq. (4a) and (4b);
8. Evaluate the longitudinal flange reinforcement ratios ρ_{f1} and ρ_{f2} by Eq. (3) and the subsequent values of A_{f1} and A_{f2}. If necessary, increase them to satisfy other conditions (for example, minimum value of flange reinforcement or given ratio A_{f1}/A_{f2}); the use of flange reinforcements higher than the minimum required is always possible because the whole procedure is based on the hypothesis of perfect plasticity; and
9. Repeat Steps 3 to 8 modifying the trial values to minimize the longitudinal flange reinforcement.

In case (b), N-M-V-ρ_f interaction domains are obtained repeating the aforementioned procedure by varying the internal actions. More commonly, normalized n-m-v-ω_f domains will be produced in which the normalized internal actions are defined as

$$n = -\frac{N}{f_{cd1}S} \qquad m = \frac{M}{f_{cd1}Sh} \qquad v = \frac{V}{f_{cd2}b_wh_w} \tag{7}$$

where S and h are area and depth of the whole cross section; b_w and h_w are width and depth of the web; and ω_f is the longitudinal flange reinforcement mechanical ratio.

As an example, the box cross section of a bridge pier has been examined (Fig. 3). According to the design practice, a

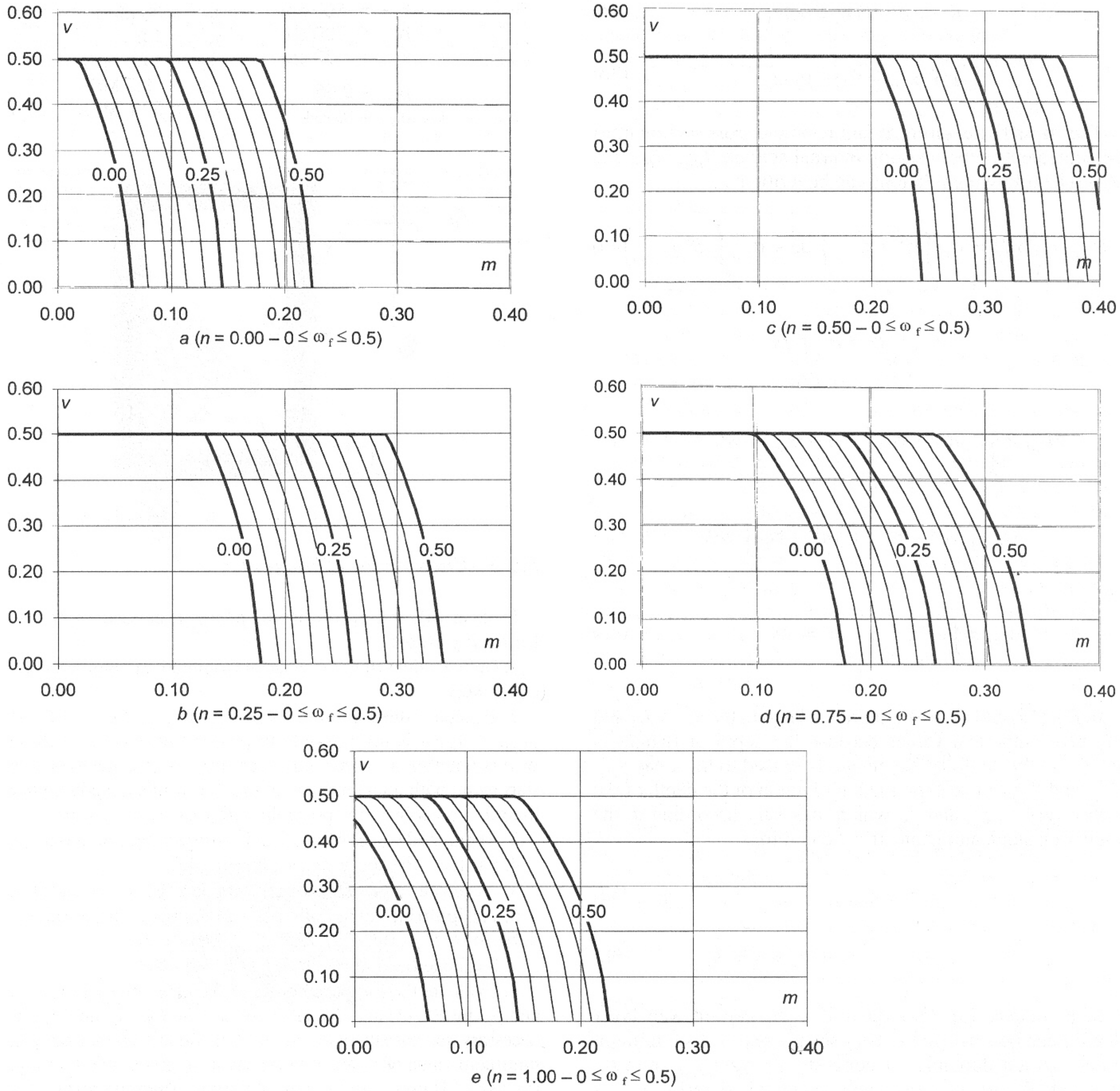

Fig. 4—v-m interaction domains for $\omega_{wt} = 0.5$ and $\omega_{wl} = 0.2$.

uniform longitudinal reinforcement has been assumed in flanges and web: $\rho_f = 0.004$; $\rho_{wl} = 0.004$, that is $\omega_{wl} = 0.2$. The transversal web reinforcement mechanical ratio ω_{wt} has been assumed equal to 0.5.

For the internal design actions $N_{Sd} = -15{,}000$ kN, $M_{Sd} =$ 45,000 kNm, and $V_{Sd} = 4500$ kN (that is $n = 0.2122$, $m =$ 0.2122, $v = 0.3157$), procedure (a) gives the additional flange reinforcement mechanical ratios $\omega_{f1} = \omega_{f2} = 0.36$ (corresponding to $\Delta A_{f1} = \Delta A_{f2} = 182$ cm^2), equal in both flanges because the bridge pier is subjected both to positive and negative bending moment.

The influence of the shear force v on the m-n resistance of the cross section, and on the necessary additional longitudinal flange reinforcement ω_f, is described in a more general way by interaction domains. Using procedure (b), the following domains have been plotted:

- *v-m* domains, for additional flange reinforcement mechanical ratio ω_f ranging from 0 to 0.5 and for given values of n (0.00, 0.25, 0.50, 0.75, 1.00), Fig. 4(a) through 4(e);
- *m-n* domains, for additional flange reinforcement mechanical ratio w_f ranging from 0 to 0.5 and for given values of v (0, 0.1, 0.2, 0.3, 0.4, 0.5), Fig. 5(a) through 5(f);
- *m-n* domains, for normalized shear force v ranging from 0 to 0.5 and for given values of additional flange reinforcement mechanical ratio ω_f (0, 0.1, 0.2, 0.3, 0.4, 0.5), Fig. 6(a) through 6(f).

The resistance domains show that:

- The presence of a normalized shear force $v \leq 0.25$ has a small influence on the bending moment capacity, for all the values of axial force and flange reinforcement;
- The presence of a larger shear force ($v > 0.25$) reduces the bending moment capacity, particularly for high values of axial force ($n > 0.5$); and

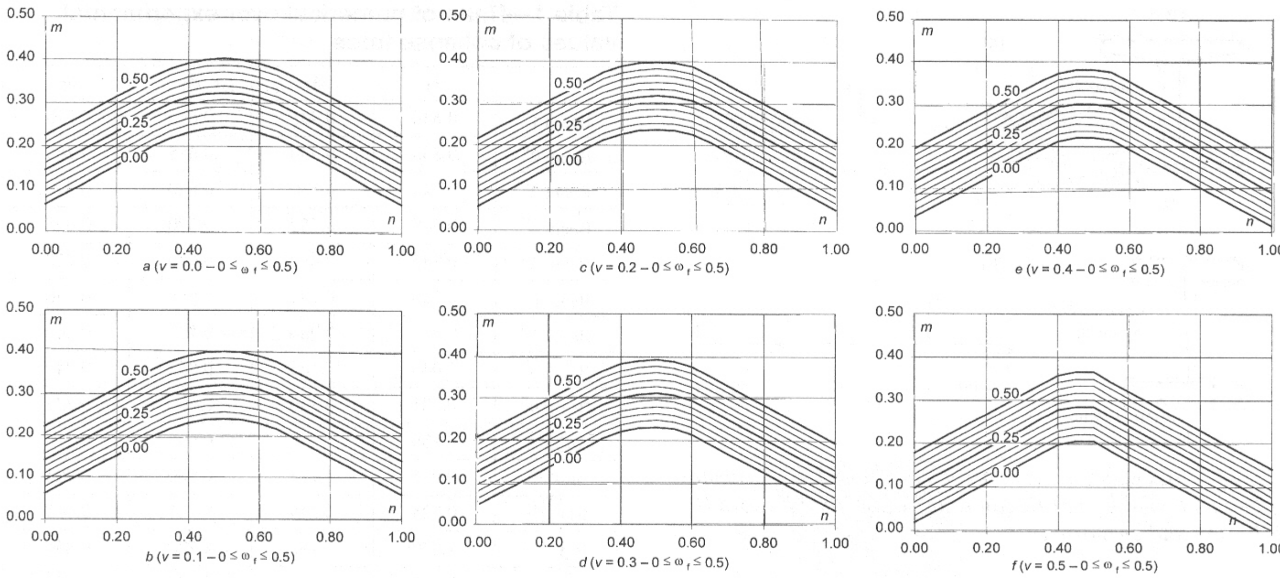

*Fig. 5—*m-n *interaction domains for given values of* v(ω_{wt} = 0.5, ω_{wl} = 0.2).

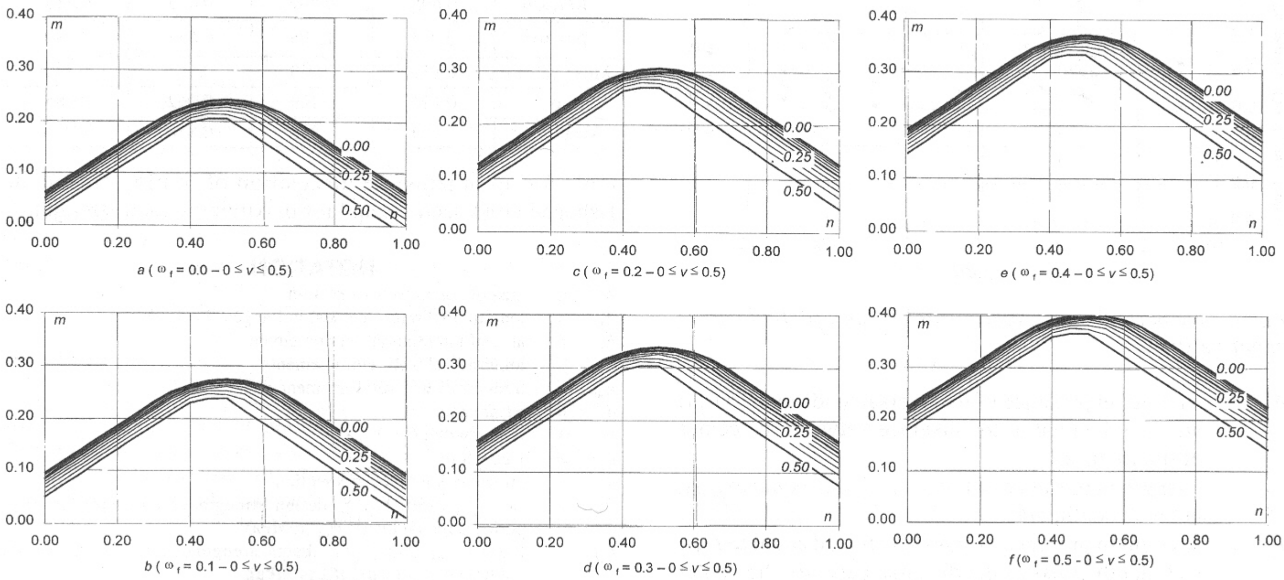

*Fig. 6—*m-n *interaction domains for given values of* ω_f (ω_{wt} = 0.5, ω_{wl} = 0.2).

- The increment of bending moment capacity due to the increasing of longitudinal flange reinforcement is scarcely dependent on the value of shear force.

Comparison with test results

The reliability of the model proposed in the previous section has been validated by comparing its numerical results to the values of ultimate strength given by failure tests[9,12] performed on thin-webbed reinforced concrete (RC) beams, made by different concrete (with strength ranging from 15 to 65 MPa). Their I-shaped cross section (Fig. 7(a) and (b)) is designed to fail in shear by crushing of the web concrete, after the stirrups yield. Geometrical dimensions and statical schemes of the beams are shown in Fig. 7(a) and (b); further details may be found in the original papers.

The comparison has been carried on by evaluating the ratio of the ultimate force values, numerically obtained by the proposed approach, over those given by the experimental evidences. In the numerical analyses, the uniaxial strength f_{cd1} has been evaluated assuming $\alpha = 1$ because no long time effect should be considered in analyzing tests performed in short time, and $\gamma_c = 1$ because no safety coefficient has to be used. Two different values have been considered for the strength in presence of transversal load: in a first case (A), it has been assumed equal to f_{cd1}, while in a second one (B), the reduced value f_{cd2} suggested by CEB-FIP Model Code 1990 has been used. A further problem is the choice of the web depth of the numerical model: in a first case (1), it has been considered equal to the net depth of the beam web, while in a second one (2), it has been assumed equal to the distance between the center of the flanges. Four numerical cases have been thus considered:

A_1 = strength in presence of transversal load equal to f_{cd1}; net web depth;

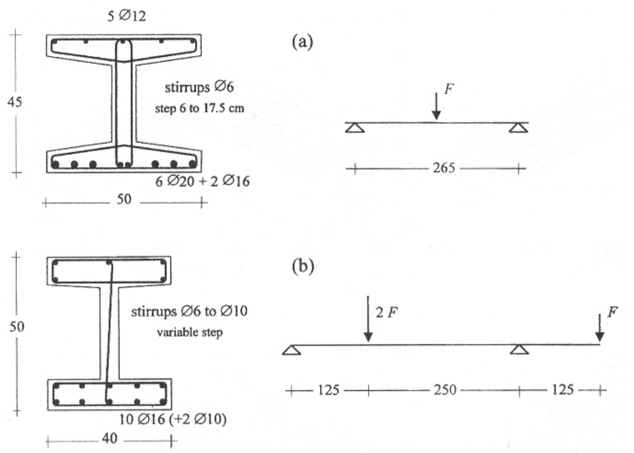

Fig. 7—Cross sections, reinforcements, and static schemes of beams tested by (a) Regan and Rezai-Jorabi[9]; and (b) Robinson and Demorieux.[12]

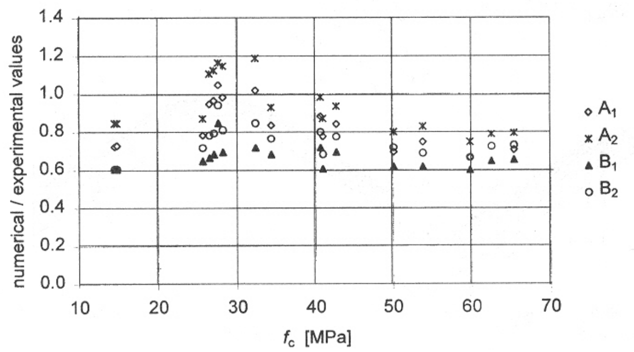

Fig. 8—Numerical over experimental values, plotted versus concrete strength.

A_2 = strength in presence of transversal load equal to f_{cd1}; web depth equal to the distance between the center of the flanges;

B_1 = strength in presence of transversal load equal to f_{cd2}; net web depth; and

B_2 = strength in presence of transversal load equal to f_{cd2}; web depth equal to the distance between the center of the flanges.

Table 1 and Fig. 8 give the ratio of numerical values over experimental values of the collapse force for all the examined beams. The mean value μ, the variance σ, and the characteristic value (μ + 3σ) have been used to statistically analyze the results. The hypothesis (A), strength in presence of transversal load equal to f_{cd1}, is not conservative because it gives characteristic values larger than unity. On the contrary, the use of the reduced value f_{cd2} suggested by CEB-FIP Model Code 1990 (B) gives a lower bound solution of the resistance domain. Finally, Fig. 8 confirms the reliability of the numerical model for all the values of concrete strength.

CONCLUSIONS

The proposed approach allows an easy evaluation of the reinforcement necessary to withstand the contemporary presence of axial force, bending moment, and shear force. Its reliability is confirmed by the comparison with the results of tests performed on I-shaped beams. The approach and the subsequent interaction domains thus appear to be powerful tools for optimizing the disposition of reinforcements in I-shaped cross sections and for maximizing their strength.

Table 1—Ratio of numerical over experimental values of collapse force

	A_1	A_2	B_1	B_2
Beam 1[9]	0.840	0.935	0.695	0.775
Beam 2[9]	0.835	0.930	0.685	0.765
Beam 3[9]	0.745	0.830	0.620	0.690
Beam 4[9]	0.665	0.750	0.600	0.665
Beam 5[9]	0.785	0.870	0.650	0.720
Beam 6[9]	0.705	0.795	0.655	0.730
Beam 7[9]	0.880	0.980	0.720	0.800
Beam 8[9]	1.045	1.165	0.845	0.940
Beam 9[9]	0.785	0.790	0.650	0.725
BQ6[12]	0.980	1.145	0.695	0.810
BQ7[12]	0.965	1.125	0.680	0.795
BQ12[12]	0.725	0.845	0.605	0.605
BQ15[12]	1.020	1.190	0.720	0.850
BQ16[12]	0.730	0.850	0.605	0.605
BQ17[12]	0.775	0.870	0.605	0.680
BQ18[12]	0.695	0.800	0.615	0.715
BQ19[12]	0.945	1.105	0.665	0.780
Mean value μ	0.831	0.940	0.665	0.744
σ	0.121	0.149	0.062	0.085
Mean + 3σ	1.194	1.388	0.850	1.000

NOTATION

A_c = area of concrete cross section
A_{fi} = longitudinal reinforcement in flanges (i = 1, 2)
A_s = area of longitudinal reinforcement
A_{wl} = longitudinal web reinforcement
A_{wt} = transversal web reinforcement
a = shear span
b_{fi} = flange width (i = 1, 2)
b_w = web width
d = effective depth of cross section
f_{cd1} = $\alpha(1 - f_{ck}/250)\, f_{ck}/\gamma_c$ design strength of concrete, for long period uniaxial load[11] (f_{ck} in MPa)
f_{cd2} = $0.6\,\alpha(1 - f_{ck}/250)\, f_{ck}/\gamma_c$ design strength of concrete, in presence of transversal load[11] (f_{ck} in MPa)
f_{ck} = characteristic strength of concrete
f_{yd} = design yield strength of steel reinforcement
h = depth of whole cross section
h_{fi} = flange depth (i = 1, 2)
h_w = web depth
M_{Rd} = resisting bending moment
M_{Sd} = design bending moment
m = normalized bending moment
N_{Rd} = resisting axial force (positive when tensile)
N_{Sd} = design axial force (positive when tensile)
n = normalized axial force (positive when compressive)
S = area of the whole cross section
S_{fi} = area of flange layers (i = 1, 2)
S_{wi} = area of web layers (i= 1, 2, 3)
s = spacing of transversal web reinforcement
V_{Rd} = resisting shear force
V_{Sd} = design shear force
v = normalized shear force
z_i = depth of layer i
z_{3min} = minimum depth of the central web layer
α = coefficient that accounts for strength reduction under long time actions
γ_c = partial safety factor for concrete
ϑ = angle of compressive stresses

ρ_{fi} = $(A_{fi})/(b_{fi}h_{fi})$ longitudinal flange reinforcement ratio (i = 1, 2)
ρ_{wl} = $(A_{wl})/(b_w h_w)$ longitudinal web reinforcement ratio
ρ_{wt} = $(A_{wt})/(b_w s)$ transversal web reinforcement ratio
σ_{fi} = axial stress in flange layers (i = 1, 2)
σ_{wi} = axial stress in web layers (i = 1, 2, 3)
τ = shear stress in the central web layer
ω_{wfi} = $(A_{fi})/(b_{fi}h_{fi})$ $(f_{yd})/(f_{cd1})$ longitudinal flange reinforcement mechanical ratio (i = 1, 2)
ω_{wl} = $(A_{wl})/(b_w h_w)(f_{yd})/(f_{cd2})$ longitudinal web reinforcement mechanical ratio
ω_{wt} = $(A_{wt})/(b_w s)(f_{yd})/(f_{cd2})$ transversal web reinforcementmechanical ratio

REFERENCES

1. Puleri, G.; Russo, G.; and Zingone, G., "Flexure-Shear Interaction Model for Longitudinally Reinforced Beams," *ACI Structural Journal*, V. 88, No. 1, Jan.-Feb. 1991, pp. 60-68.
2. Puleri, G., and Russo, G., "Stirrup Effectiveness in Reinforced Concrete Beams under Flexure and Shear," ACI JOURNAL, *Proceedings* V. 94, No. 3, May-June 1997, pp. 227-238.
3. Nielsen, M. P.; Braestrup, M. W.; and Bach, F., "*Rational Analysis of Shear in Reinforced Concrete Beams,*" IABSE Proceedings, P-15/78, 1978.
4. Mancini, G.; Recupero, A.; and Scilipoti, C. D., "Shear in RC Elements with Longitudinal and Transversal Web Reinforcements" ("Taglio Nel C.A. in Presenza di Armature d'Anima Longitudinali e Erasversali"), *Studi e Ricerche*, Scuola di Specializzazione in Costruzioni in C.A., Fratelli Pesenti, Politecnico di Milano, V. 17, 1996. (in Italian)
5. Fillo, L., and Halvonik, J., "Ultimate Limit State Design of Reinforced Concrete Members Subjected to Bending, Axial Load and Shear," *Bulletin d'Information*, CEB, No. 223, June 1995.
6. Fanti, G., and Mancini, G., "Shear, Normal Force, Bending Moment Interaction in Bridge Piers," *Proceedings of the 1st Japan-Italy Workshop on Seismic Design and Retrofit of Bridges*, Tsukuba, Japan, Mar. 13-14, 1995.
7. Mancini, G., and Recupero, A., "Combined Flexural and Shear Effects on Bridge Piers," *Proceedings of the 2nd Italy-Japan Workshop on Seismic Design and Retrofit of Bridges*, Rome, Feb. 27-28, 1997.
8. Mancini, G., and Recupero, A., "Interaction of Axial Force, Bending Moment and Shear Force in RC Structures" ("Interazione tra Azione Assiale, Momento Flettente e Taglio Nelle Strutture in C.A."), *Studi e Ricerche*, Scuola di Specializzazione in Costruzioni in C.A., Fratelli Pesenti, Politecnico di Milano, V. 20, 1999. (in Italian)
9. Regan, P., and Rezai-Jorabi, H., "The Shear Resistance of Reinforced Concrete I-Beams," *Studi e Ricerche*, Scuola di Specializzazione in Costruzioni in C.A., Fratelli Pesenti, Politecnico di Milano, V. 9, 1987.
10. Mancini, G.; Napoli, P.; and Recupero, A., "Shear-Transverse Bending Interaction in the Webs of RC/PC Box Girder," *5th International Conference on Short and Medium Span Bridges*, Calgary, Alberta, Canada, July 13-16, 1998.
11. CEB-FIP Model Code 1990, Thomas Telford, ed., 1993.
12. Robinson, J. R., and Demorieux, J. M., "Essais de Poutres en Double té en Béton Armé," *Annales de l'ITBTP*, Beton, No. 153, Paris, Jan. 1976.

APPENDIX

Equilibrium conditions: central web layer

Consider a portion of the central web layer cut by a plane parallel to the concrete stress field, that is, with a ϑ degrees angle to the longitudinal axis of the structural element (Fig. 9(a)). For the equilibrium conditions along the axis and in the transverse direction, is

$$\rho_{wl}\sin\vartheta\sigma_{swl} - \tau\cos\vartheta - \sigma_{w3}\sin\vartheta = 0 \quad (8a)$$

$$\rho_{wt}\cos\vartheta\sigma_{swt} - \tau\sin\vartheta = 0 \quad (8b)$$

For the equilibrium condition in the transverse direction to the portion of the central web layer cut by a plane orthogonal to the concrete stress field (Fig. 9(b)), is

$$\rho_{wl}\sin\vartheta\sigma_{swt} + \tau\cos\vartheta - \sigma_c\sin\vartheta = 0 \quad (8c)$$

Equations (8a) and (8b) may be written as

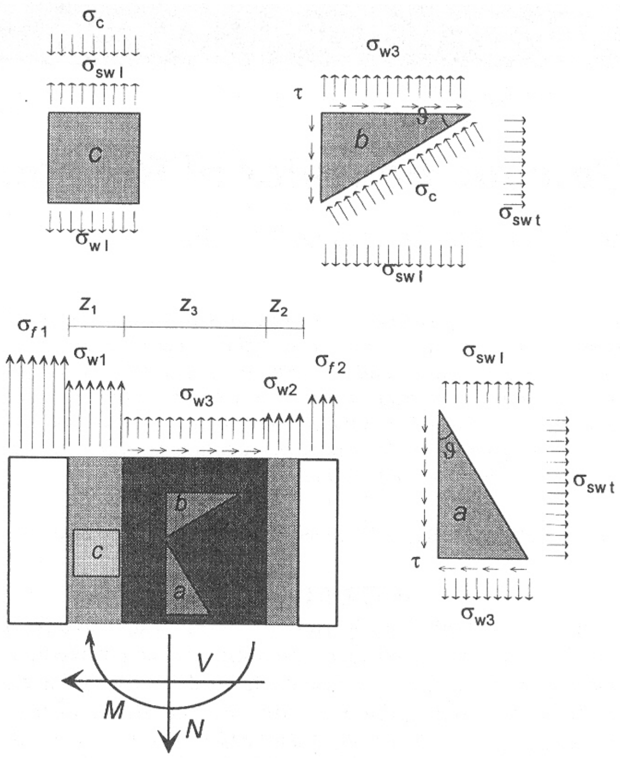

Fig. 9—Stresses acting on portions of layers.

$$\tau = (\rho_{wl}\sigma_{swl} - \sigma_{w3})\tan\vartheta \quad (9a)$$

$$\tau = \rho_{wt}\cot\vartheta\,\sigma_{swt} \quad (9b)$$

Furthermore, evaluating from Eq. (8b) also $\rho_{wt}\sigma_{swt} = \tau\tan\vartheta$ and substituting this into Eq. (8c) gives

$$\tau = \sigma_c\sin\vartheta\cos\vartheta \quad (9c)$$

Finally, by imposing that

$$|\sigma_{swl}| \le f_{yd}, \sigma_{swt} \le f_{yd} \text{ and } \sigma_c \le f_{c2},$$

Eq. (1a) to (1c) are obtained.

Equilibrium conditions: lateral web layers and flange layers

Consider a portion of the first lateral web layer cut by planes parallel and orthogonal to the longitudinal axis of the structural element (Fig. 9(c)). Imposing the equilibrium conditions along the axis gives

$$\sigma_{wl} = \rho_{wl}\sigma_{swl} - \sigma_c \quad (10)$$

By imposing that

$$|\sigma_{swl}| \le f_{yd} \text{ and } 0 \le \sigma_c \le f_{c1},$$

Eq. (2a) is obtained. In the same way, Eq. (2b), (3a), and (3b) are obtained.